

# Magnetic field direction and phase deviation effects on heat transfer for MHD natural convection of CNT-water nanofluid

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**Abstract** — In this paper, the effects of magnetic field direction and phase deviation angles on heat transfer of MHD natural convection of Carbon NanoTube (CNT)-water nanofluid in square cavity heated sinusoidally from bellow have been analyzed. The first part of the study has been carried out for the pertinent parameters in the following ranges: the magnetic field direction,  $\phi= 0-180^\circ$ , the phase deviation,  $\chi=0-90$  for Rayleigh number  $Ra=10^5$ , Hartmann number  $Ha=15$  and for a volume fraction nanoparticles equal to 4%. For the second part, the Rayleigh number has been varied from  $2.5 \times 10^4$  to  $12.5 \times 10^4$  while the remaining parameters are considered to be constant. Results show a nonlinear behavior of the average Nusselt number ( $Nu_{avg}$ ) for fixed phase deviation ( $\chi=cte$ ). Furthermore, the negative role of magnetic field direction in ameliorating the heat transfer rate was deduced. Finally, correlations which define the relation between Rayleigh number and the average Nusselt number was established.

**Keywords**—Heat transfer, Magnetoconvection, Nanofluids, Sinusoidal boundary condition, Magnetic field direction, Phase deviation angle.

## I. INTRODUCTION

Applying a magnetic field to an electrically conducting fluid, named magneto-hydrodynamic (MHD), for the natural convection in enclosure cavity under varying thermal boundary condition becomes one of most studied topics in research due to its various applications in industrial, technological and medical application such as solar technology, cooling of nuclear reactor , crystal growth in liquid and microelectronics device , etc. Moreover, adding nanoparticles to the based fluid, which known as nanofluid, makes it more important in these applications. This importance attracts us to consider the present problem. Many parameters affect the heat transfer of MHD natural convection of nanofluids in enclosure cavity such as type of nanoparticle [1], Hartmann number [2], volume fraction nanoparticles [3], Rayleigh number [4] and thermal boundary condition [5]

The present numerical study aims to study, in view of engineering applications, the MHD natural convection of CNT-water nanofluids in square cavity heated sinusoidally from below. The heat transfer will be evaluated, in the first part, under different values of magnetic field direction ( $\phi$ ) and phase deviation ( $\chi$ ) for given values of Rayleigh number ( $Ra$ ),

Hartmann number ( $Ha$ ) and volume fraction nanoparticles ( $\Phi$ ). The second part will be devoted to heat transfer correlations. In this part, a relation between the average Nusselt number and Rayleigh number will be established.

## II. PHYSICAL MODEL AND NUMERICAL APPROACH

The physical model of the considered work is presented in fig.1. It consists of a square cavity with dimension  $H$ . This cavity is filled with water based nanofluids containing nanoparticles of carbon nanotube (CNT), which is considered Newtonian and incompressible. The vertical walls are assumed to be thermally adiabatic. A sinusoidal temperature distribution, according to the space coordinate ( $x$ ) with a phase deviation  $\chi$ , is applied to the bottom wall while the top wall is isothermally cooled. In Addition, the base fluid and the CNT nanoparticles are in thermally equilibrium. The thermophysical properties of the nanofluid, reported in table.1[6], are assumed to be constant except for the density variation which approximated by the standard Boussinesq model. A uniform magnetic field with a constant magnitude  $B_0$  is applied under an angle  $\phi$  to the horizontal. Moreover, the induced magnetic field produced by the motion of the electrically conducting nanofluid is negligible compared to the applied magnetic field. In addition, it assumed that the viscous dissipation due to the Joule heating is neglected. Furthermore, the radiation heat exchange is considered negligible. The dynamic viscosity and the thermal conductivity of nanofluid containing a dilute suspension of small rigid spherical particles are modeled according to Brinkman [7] and Maxwell [8] models. Also the electrical conductivity is defined according to the Maxwell model [9].

Based on the above assumptions, the governing dimensionless equations for conservation of mass, momentum, and energy of the two-dimensional unsteady nanofluids magnetoconvection (MHD) flow can be written as follows:

Conservation of mass equation:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

Momentum equation:

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_{nf}} \frac{1}{\alpha_f} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - Ha^2 Pr \frac{\sigma_{nf}}{\sigma_f} \frac{\rho_f}{\rho_{nf}} \sin(\varphi) (U \sin(\varphi) + V \cos(\varphi)) \quad (2)$$

$$\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{(\rho\beta)_{nf}}{\rho_{nf} \beta_f} Ra Pr \theta + \frac{\mu_{nf}}{\rho_{nf}} \frac{1}{\alpha_f} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - Ha^2 Pr \frac{\sigma_{nf}}{\sigma_f} \frac{\rho_f}{\rho_{nf}} \cos(\varphi) (V \cos(\varphi) + U \sin(\varphi)) \quad (3)$$

Energy equation:

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (4)$$

The dimensionless parameters appearing in the above equations are the Rayleigh number  $Ra$ , the Prandtl number  $Pr$ , and the Hartmann number  $Ha$  which are defined as:

$$Ra = \frac{g(T_H - T_C)H^3 \beta_f}{\nu \alpha}; \quad Pr = \frac{\nu_f}{\alpha_f}; \quad Ha = HB_0 \sqrt{\frac{\sigma_f}{\mu_f}}$$

The average Nusselt number ( $Nu_{avg}$ ) is defined in the heated wall as:

$$Nu_{avg} = -\frac{K_{nf}}{K_f} \int_0^1 \frac{\partial \theta}{\partial Y} \Big|_{Y=0} dX$$

It was obtained by integrating the  $\frac{\partial \theta}{\partial Y}$  along three walls nodes to ensure accuracy.

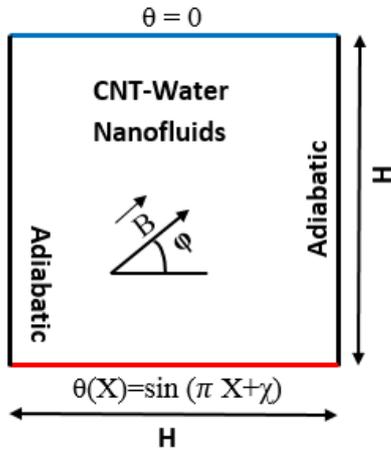


Fig.1 Physical model

### III. RESULT AND DISCUSSION

The presented results in this work are obtained by using a finite volume home FORTRAN code, named NASIM and developed by the second author which use multi-grid solver explained in details in previous works [10].

#### A. Magnetic field direction and phase deviation angle effects on the heat transfer

In this section we investigate the magnetic field direction ( $\varphi$ ) and the phase deviation ( $\chi$ ) effects on the heat transfer rate of MHD natural convection of CNT-water nanofluids at  $Ra=10^5$ ,  $\Phi=0.04$  and  $Ha=15$ .

TABLE 1  
THERMOPHYSICAL PROPERTIES OF CNT

	Pure water	CNT
$\rho(Kg\ m^{-3})$	997.1	1350
$C_p(JKg^{-1}K^{-1})$	4179	650
$K(W\ m^{-1}\ K^{-1})$	0.613	3500
$\beta(K^{-1})$	$21 \times 10^{-5}$	$4 \times 10^{-5}$
$\sigma(\Omega^{-1}\ m^{-1})$	0.05	$5 \times 10^6$

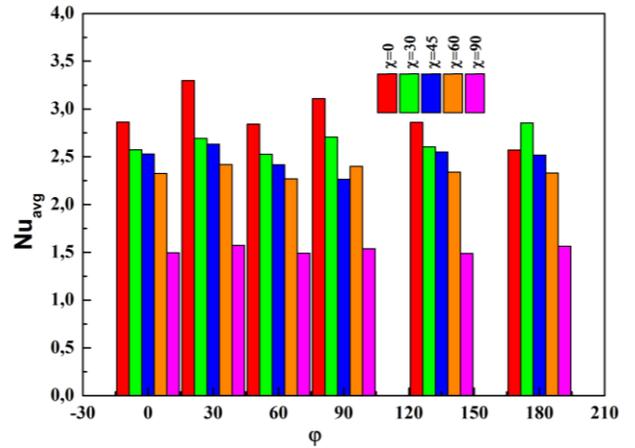
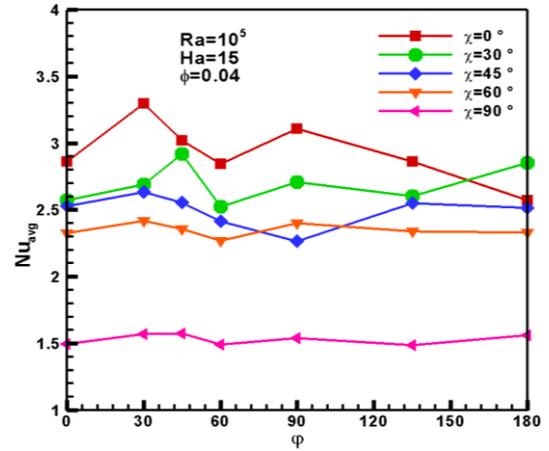


Fig.2. (a) Variation of average Nusselt number ( $Nu_{avg}$ ) in terms of magnetic field orientation ( $\varphi$ ) for different phase deviations ( $\chi$ ) for  $Ra=10^5$ ,  $\Phi=0.04$  and  $Ha=15$ . (b) Histogram

In this application the temperature distribution at the bottom wall changes among the X coordinate. That is why it is important to study the heat transfer rate at this wall.

Fig. 2a. illustrates the variations of the average Nusselt number in terms of the magnetic field direction ( $\phi$ ) at different phase deviation ( $\chi$ ). Results show that generally for a given magnetic field direction ( $\phi=cte$ ), the phase deviation ( $\chi$ ) plays a negative role in ameliorating heat transfer where the average Nusselt number decrease with the growth of the phase deviation from 0 to 90. This effect can be explained by the variation of the heating and cooling locations. In addition a non-linear behavior of the average Nusselt number, for fixed phase deviation ( $\chi=cte$ ), is observed. Moreover we found, in the majority of cases, that the most heat transfer is obtained for  $\chi=0$  for different  $\phi$  values. Furthermore, for  $\chi=0, 45$  and  $60$  the heat transfer rate achieves its maximum for  $\phi=30$  however  $\phi=45$  gives the maximum heat transfer for  $\chi=30$  and  $90$ .

These results were confirmed by the histogram presented in fig. 2b. This histogram is another way to demonstrate these combined effects in order to better control them. Finally, it is noticeable that the optimal case of the heat transfer rate is obtained for ( $\phi=30, \chi=0$ ).

Fig. 3 exhibits map contours of the heat transfer rate for MHD natural convection of CNT-water nanofluids in a square cavity, heated sinusoidally from below, under any magnetic field orientation between 0 and 180 and any phase deviation between 0 and 90. This map describes the heat transfer rate at the heated wall. It was obtained by using 2D quadratic polynomial interpolation. This map is an effective way to control variation of the average Nusselt number by varying  $\phi$  and  $\chi$ , so for any given values ( $\phi, \chi$ ) we will be able to estimate the corresponding transfer rate.

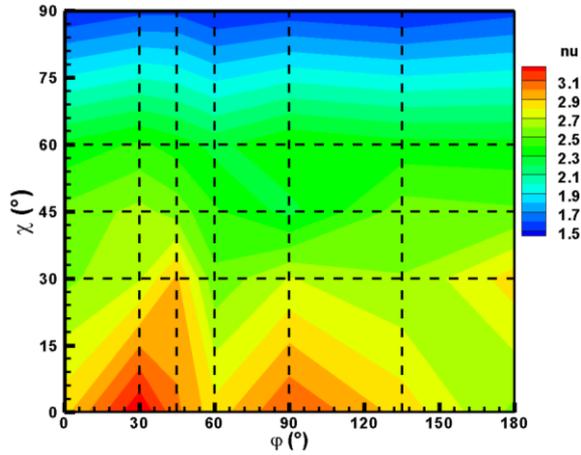


Fig. 3. Mapping of the heat transfer rate of MHD natural convection of CNT-water under different magnetic field direction ( $\phi$ ) and phase deviations ( $\chi$ ) for  $Ra=10^5, \Phi=0.04$  and  $Ha=15$ .

### B. Heat transfer correlation

In this part, attention is focused on the Rayleigh number effects in heat transfer rate. The magnetic field direction,  $\phi$ , is fixed to 30 and the phase deviation,  $\chi$ , is maintained equal to 0.

The corresponding optimal heat transfer case is previously deduced according to the precedent paragraph. The Rayleigh number ranges from  $2.5 \times 10^4$  to  $1.25 \times 10^5$ . The Hartmann number and the volume fraction nanoparticles have the same value as the first part of calculation ( $\Phi=0.04$  and  $Ha=15$ ).

TABLE 2  
BENCHMARK SOLUTION

Ra	$Nu_{avg}$
$2.5 \times 10^4$	1.630
$5 \times 10^4$	2.514
$7.5 \times 10^4$	2.984
$10 \times 10^4$	3.298
$12.5 \times 10^4$	3.535

Fig. 4 demonstrates the effects of Rayleigh number  $Ra$  on the average Nusselt number. As it can be observed in the table.2, increasing the Rayleigh number  $Ra$  leads to an increase in the heat transfer rate. Fig 5.a exhibits the variation of average Nusselt number as a function of  $\text{Log}(Ra)$ .

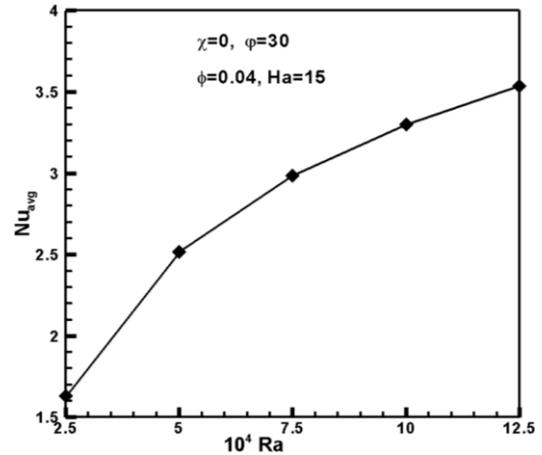


Fig. 4. Variation of average Nusselt number in terms of Rayleigh number for  $\phi=30, \chi=0, \Phi=0.04$  and  $Ha=15$

A linear progression trend for  $Nu_{avg}$  appears. Thus the correlation defines the relation between  $Nu_{avg}$  and  $Ra$  corresponding to the MHD natural convection of CNT-water nanofluids under a magnetic field direction equal to 30 for deviation phase equal to 0,  $\Phi=0.04$  and  $Ha=15$  is given by the following equation:

$$Nu_{avg} = -10.348 + 1.185 \text{Log}(Ra)$$

In order to test the capacity of the correlation established, the obtained numerical data results were compared with the values obtained from correlation. As shown in Fig 5.b the trend of the obtained values from correlation and the measured data are usually the same. In addition, a slight variation was acquired between them.

This variation, as presented in Fig.6, leads to a relative standard deviation (SD) equal to 3.378%. Hence the results computed using the numerical simulation and the correlation

are in reasonable agreement. Therefore we conclude that the correlation results may be used in future numerical simulations.

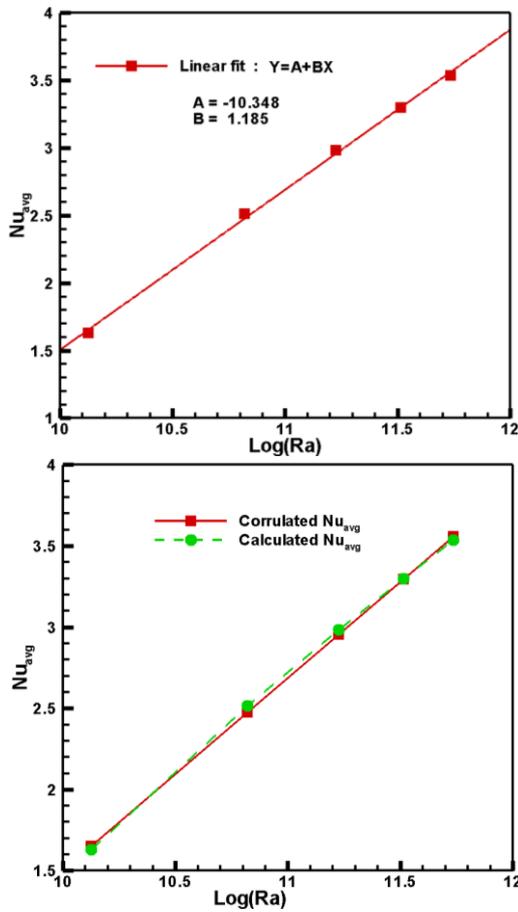


Fig.5. (a) Variation of average Nusselt number versus Log (Ra) (b) Variation of numerical and correlated average Nusselt number versus Log (Ra) at  $\phi=30$ ,  $\chi=0$ ,  $\Phi=0.04$  and  $Ha=15$ .

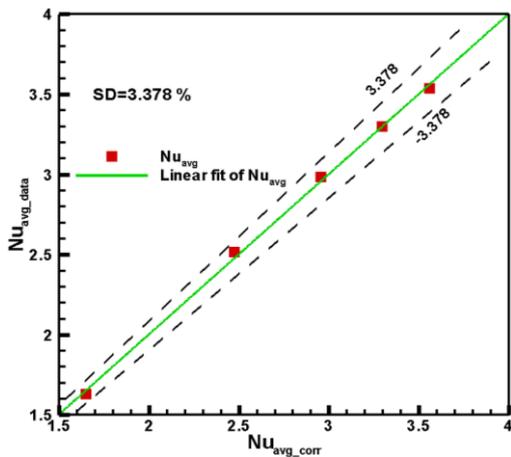


Fig. 6. Comparison between numerical and correlated average Nusselt number.

#### IV. CONCLUSIONS

In the present study, the effects of both magnetic field direction and phase deviation on the heat transfer rate for

MHD natural convection of CNT-water nanofluid within square cavity heated sinusoidally from below, for  $Ha=15$  and  $\Phi=0.04$ , have been investigated. In addition, a relation between the Rayleigh number and the average Nusselt number have been established.

The first considered case is about the effects of phase deviation ( $\chi$ ) and magnetic field direction ( $\phi$ ) on heat transfer for  $Ra=105$ . Some important points can be deduced from the obtained results such as:

- For  $\phi=cste$ , a negative role was played by the phase deviation on ameliorating the heat transfer.
- For  $\chi=cste$ , a non-linear behavior of the average Nusselt number is observed.
- The most heat transfer is obtained for  $\chi=0$  for different  $\phi$  values in the majority of cases.
- The optimal case of heat transfer is obtained for  $\phi=30$ ,  $\chi=0$

In the second stage, correlation defines the relation between the Rayleigh number and the average Nusselt number, was deduced. References

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